Seminar of Computer Networks:
Online Social Networks and Network Economics
Prof. Guido Schäfer

Homework 2

Due date: June 24, 2011

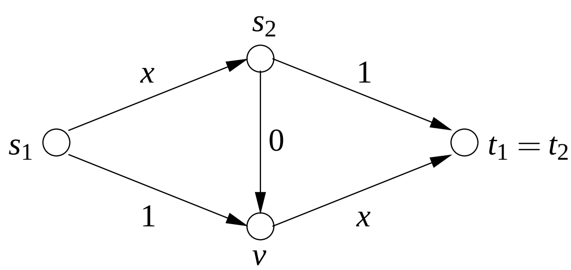
You can do the homework on your own or in groups of two. In the latter case, please hand-in **one** solution set per group and indicate the names of the group members. Please make sure that the solutions are clearly written.

Please note that some of the problems are marked as **optional**. You are requested to solve **all** problems that are **non-optional** and **one** problem that is marked **optional**. That is, **altogether** you are asked to solve **4 problems**.

Please **hand-in** your solutions by **June 24, 2011** and **keep a copy for yourself**. You can hand-in your solutions in **office B118**.

We will post the solutions to this homework after the due date and might ask you about your mistakes in the final exam.

Problem 1. Consider the following selfish routing instance:



The arcs are labeled with their respective latency functions. Assume that one unit of flow is to be sent from s_i to t_i, i.e., $r_i = 1$ for $i \in \{1,2\}$.

(a) Determine a Wardrop flow (see Definition 2.3 of Lecture Notes) and its respective cost.

(b) Compute an optimal flow and its respective cost. (*Hint:* Use the characterization of optimal flows given in Corollary 2.4 of the Lecture Notes.)

Problem 2. Show that the price of anarchy of selfish routing games is one if all latency functions are *monomials* of degree $d \in \mathbb{N}$, i.e., for every arc $a \in A$, $\ell_a(x) = p_a x^d$ for some $p_a \geq 0$. (*Hint:* Corollary 2.4 of the Lecture Notes might be helpful here.)

Problem 3. Consider the single-item auction setting introduced in Section 5.1 of the Lecture Notes.

(a) In a *first-price auction* the item is given to a player whose bid is largest (ties are broken arbitrarily) at a price equal to the bid of this player. Show that the first-price auction is not strategyproof.

(b) Show that in a second-price auction a player might be strictly worse-off if he does not bid truthfully. That is, show that for every player $i \in N$ and for every bid $b_i \neq v_i$ there is a bidding profile b_{-i} of the other players such that $u_i(b_{-i}, b_i) < u_i(b_{-i}, v_i)$.

Problem 4 (optional). Consider the combinatorial auction setting introduced in Section 5.2 of the Lecture Notes. Suppose we are given a function h_i for every player $i \in N$ that assigns an arbitrary real value to every bidding profile b_{-i} of the other players (i.e., h_i does not depend on the bid of player i). Consider the adapted version of the VCG mechanism (see Algorithm 1).

Algorithm 1 Adapted VCG mechanism

Collect the bids $(b_i(S))$ for every player $i \in N$ and every set $S \subseteq M$.

Choose an allocation $a^* \in O$ such that

$$a^* = \arg\max_{a \in O} \sum_{i \in N} b_i(a).$$

Compute the payment p_i of player $i \in N$ as

$$p_i = h_i(b_{-i}) - \sum_{j \in N, j \neq i} b_j(a^*).$$

Return the allocation a^* and the payments $(p_i)_{i \in N}$.

(a) Prove that this adapted version of the VCG mechanism is strategyproof.

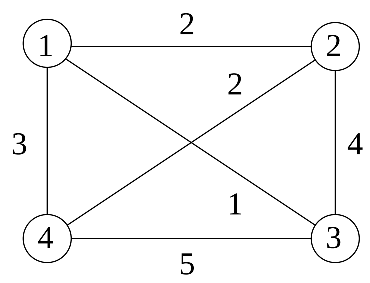
(b) Determine the functions $(h_i)_{i \in N}$ that yield the payments of the VCG mechanism which was introduced in the lecture.

Problem 5 (optional). In the *max-cut game* we are given an undirected graph $G = (N, A)$ on the player set $N = [n]$ and non-negative arc-weights $(w_a)_{a \in A}$. Every player $i \in N$ chooses one of two possible colors for his vertex: *white* ($x_i = 1$) or *black* ($x_i = 0$). A strategy profile $x = (x_1, \ldots, x_n)$ thus defines a partition $(S, N \setminus S)$ of N into white and black vertices, respectively. Define $\delta(S) = \{\{i, j\} \in A \mid i \in S, j \in N \setminus S\}$ as the set of arcs whose endpoints have different colors. The goal of each player $i \in N$ is to choose a color such that the total weight of all incident arcs that have differently colored endpoints is maximized; more formally, the utility of player i for a given strategy profile S is

$$u_i(S) = \sum_{\{i,j\} \in \delta(S)} w_{\{i,j\}}.$$

Suppose the social cost function $c(S)$ for a given strategy profile S is defined as $c(S) = \sum_{a \in \delta(S)} w_a$.

(a) Show that for the following instance there are two Nash equilibria having different social costs.



(b) Show that max-cut games always have pure Nash equilibria. (*Hint:* Study the effect of single-player deviations on the sum of the utilities of all players.)