Seminars of Computer Networks
Academic year 2009/2010
Second Homework
Deadline: June 20, 2010

Stefano Leonardi

Question 1.

I. Consider the problem of auctioning k identical items to $n \geq k$ bidders. Bidder i has valuation v_i for receiving one item. The generalized Vickrey auction allocates the k items to the k highest bidders at the price of the $k + 1$th highest bid.

1. Show that this auction is also the result of the application of VCG.

II. Consider a graph $G = (V, E)$ with every edge $e \in E$ controlled by an independent agent with non-positive valuation $c(e) = -v(e)$. You can see $c(e)$ as the cost incurred by the agent for establishing the connection. The set of alternatives for the mechanism is formed by all paths in the graph from a source node s to a destination node t.

1. Which is the allocation produced by the VCG mechanism for this game.

2. Which are the payments for the agents.

   Observe that in this case since the valuations are negative also the payments are negative, i.e., the mechanism pays the agents that are selected in the allocation for establishing the connections.

Question 2.

   Consider the problem of auctioning k different goods to n bidders. Bidder $i = 1, \ldots, n$ has valuation v_{ij} for item $j = 1, \ldots, k$. The seller has a reserve price r_j for item j. Every bidder gets at most 1 item in any allocation.

1. Model the problem of auctioning k different goods with reserve prices by introducing a fictitious bidder for each item.

2. Define the VCG mechanism (allocation and payments) for this auction.

3. Show that any item sold to a real bidder is priced with at last the reserve price.

Question 3.

   Consider the GSP auction for 2 positions and 2 bidders. It is well known that GSP does not maximize social welfare since it is not VCG.

1. Consider the case of CTR $\alpha_1 = 1$ and $\alpha_2 = r < 1$ and valuations $v_1 = 1$ and $v_2 = 0$. Show a Nash equilibrium of the strategies of the two bidders with social welfare equal to r. How does it compare to the optimum social welfare?

2. Consider the case of bidders that are only allowed strategies (bids) $b_i \leq v_i$. Show that in general (any CTRs of the two slots and valuations of the two agents) the social welfare achieved from GSP is at least $4/5$ the optimal social welfare.

Question 4.

Show that the solution for the similarity ranking algorithm for bipartite graphs exists and it is unique: Query Rewriting through Link Analysis of the Click Graph: Atoanellis et al., In proc of VLDB 2008.