Seminar of Computer Networks

Homework 1

You can do the homeworks alone or in groups of two students. Write a single solution with both names.

Make sure that the solutions are typewritten or clear to read.

Hand in your solutions and keep a copy for yourself. After the due date we will post the solutions and in the final exam we will ask you to explain us what were your mistakes.

Due: 28/4/2010, before the class.

Problem 1. We roll two standard six-sided dice. Find the probability of the following events, assuming that the outcomes of the rolls are independent.

1. The two dice show the same number.
2. The number that appears on the first die is larger than the number on the second.
3. The sum of the dice outcomes is even.
4. The product of the dice outcomes is even.
5. The product of the dice outcomes is a perfect square.

Problem 2. Consider the Erdős-Rényi $G_{n,p}$ random-graph model.

1. What is the probability that a graph created according to $G_{n,p}$ contains only one triangle (three nodes connected with each other and no other edges exist)?
2. What is the probability that all the n nodes are connected in a line (with no other edges present)?

Problem 3. Show that for the Watts-Strogatz small-world model for $p = 0$ and as $k, n \to \infty$ the clustering coefficient approaches 3/4.

Problem 4. In the class we showed that given a directed graph $G = (V, E)$, the function $f(S) =$ "number of nodes reachable from S", for $S \subseteq V$, is submodular. Show that finding the set S that maximizes the number of nodes reachable from S subject to the constraint $|S| \leq k$ is NP-hard.

Problem 5. Consider a graph $G = (V, E)$, with $V = V_1 \cup V_2 \cup V_3 \cup V_4$, each $|V_i| = n'$, and all the V_is disjoint, so that $|V| = n = 4n'$. Assume that for each V_i a fraction p' of the edges exist, that is, $m' = p' \cdot \binom{n'}{2}$ out of the $\binom{n'}{2}$ edges within V_i exist, and that for each V_i, V_j with $i \neq j$ a fraction p'' of edges exist (i.e., $m'' = p'' \cdot n'^2$ out of the n'^2 possible edges exist). Consider the following two partitionings of the graph:

1. There are four partitions, with every V_i in each own partition
2. There are two partitions, V_1 and V_2 in one and V_3 and V_4 in the other.

Show that the modularity of the first partition is asymptotically higher from the modularity of the second if and only if $p' > p''$. Explain whether this result is expected.