Competitive Influence in Social Networks: Convergence, Submodularity, and Competition Effects*

(Extended Abstract)

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ABSTRACT

In the last 10 years, a vast amount of scientific literature has studied the problem of influence maximization. Yet, only very recently have scientists started considering the more realistic case in which competing entities try to expand their market and maximize their share via viral marketing. Goyal and Kearns [STOC 2012] present a model for the diffusion of two competing alternatives in a social network, which consists of two phases: one for the activation, in which nodes choose whether to adopt any of the two alternatives or none of them, and one for the selection, which is for choosing which of the two alternatives to adopt.

In this work we consider this two-phase model, by composing some of the most known dynamics (threshold, voter, and logit models), and we ask the following questions: (1) How is the stationary distribution of the composition of these dynamics related to those of the single composing dynamics? (2) Does the number of adopters of one of the alternatives increase in a monotone and submodular way with respect to the set of initial adopters of that alternative? (3) To what extent does the competition among alternatives affect the total number of agents adopting one of the alternatives?

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*; J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economy*

Keywords

Social Networks, Influence Diffusion, Submodularity, Voter model, Threshold model, Logit Dynamics

*This research was partially supported by the Google Focused Research Award "Algorithms for Large-Scale Data Analysis", and the EU projects FET MULTIPLEX 317532, FET SIMPOL 610704, and ERC PAAI 259515.

†Research has been conducted when this author was at Sapienza University of Rome.

Appears in: *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2015), Bordini, Elkind, Weiss, Yolum (eds.), May 4–8, 2015, Istanbul, Turkey.*

1. INTRODUCTION

In many Multi-Agent Systems (MAS) agents usually interact with and are influenced by a small set of other agents. *Social networks*, describing the interconnections among the different components of the system, become in this way a central tool for understanding how a multi-agent system evolves and how we can influence this evolution.

One of the main research issues concerning social networks is the analysis of how behavior propagates among the agents. This underlies different well-studied problems, such as *viral marketing* and *opinion formation*. Nowadays, viral marketing [16], which is the strategy of trying to reach as many users for a product/service by a word-of-mouth promotion, turns out to be an extremely prevalent topic because of the boom of social media. In the setting of opinion formation, the goal is to understand how an opinion diffuses in social networks and how a single agent adapts its own beliefs in response to the opinions of "friends." Literature on this topics abounds both in AI [15, 17] and in CS at large [1, 3, 13], as well as sociology, economics, physics, and epidemiology.

Many dynamics have been introduced in these works for modeling the evolution of the behavior of agents in the network. Among these dynamics, there are some, such as the *general threshold model* (GT) [11], the *voter model* (VOT) [6], and the *logit model* (LOG) [4], which have attracted the focus of a very large amount of literature. The typical problem in this setting is the *influence-maximization problem*, which is the problem of choosing a set of initial nodes in the network, called *seed*, whose action is fixed, such that their influence on the remaining nodes is maximized (see, for example, [11, 12, 7, 19]).

Very few works in the existing vast literature assume the presence of different alternative *products* (different goods or services or divergent opinions) competing over the network. This competition raises questions of a game-theoretic nature. Indeed, we can model the competition as a game: Players are the promoters of products, their strategies correspond to the possible choices for a set of seeds, and their utility is given by the number of nodes adopting, at the end of the evolution process, their good/service/opinion. Within this game-theoretic framework, three main problems have been considered: several works [5, 19, 8] focus on the problem of approximating the best-response strategies, that is the set of seeds maximizing the adopters of the promoted product given the other promoters' seeds. Alon et al. [2] and Tzoumas et al. [18] characterized the cases in which these games admit a Nash Equilibrium. Finally, some recent works [18, 9, 10] evaluate the performance of these equilibria with respect to the total number of active nodes.

Among these work, the one of Goyal and Kearns [9] is particularly interesting, as it proposes a two-phase model for competitive influence in social networks. This model composes of two dynamics: one, called *activation rule* (AR), models the activation of nodes, that is, how nodes decide whether to adopt a product or not, whereas the other one, named *selection rule* (SR), models the selection of one of the alternatives. However, the two-phase model of [9] does not consider any of the well-known dynamics described above. Here we study whether the results of [9] continue to hold when we compose the threshold, voter, and logit models.

Our Results. First, we study the convergence properties of these composed dynamics. Specifically, we ask whether the distribution to which the composed dynamics converges is the product of the distributions to which the composing dynamics converge. We summarize our findings in Table 1(a).

Understanding where and how the dynamics converge is a fundamental requirement for addressing more complex questions in the framework of competitive influence maximization. Within this framework, we first consider the issue of computing and approximating the best response of a promoter. Very few algorithms are known for computing a seed with bounded approximation guarantees with respect to the number of final adopters. The greedy algorithm [14] is one of these and it is known to work for any dynamics as long as the (expected) number of adopters of some product is a monotone and submodular function of the seed for that product. Goyal and Kearns [9] proved that these properties hold for the dynamics they considered. Here, we ask whether the greedy algorithm works when the threshold, voter and logit dynamics are composed; we summarize the results in Table 1(b).

Finally, we ask whether the expected number of adopters resulting from seeds placed by competing promoters is within a constant factor from the expected number of adopters that we should have if the seeds was optimally placed by a monopolist (i.e., we ask whether the price of anarchy is constant). Goyal and Kearns [9] give a positive answer for their model. Our results are summarized in Table 1(c).

		SR	
		VOT	LOG
AR	GT	Yes	Yes
	LOG	No	Yes

(a)

		SR	
		VOT	LOG
AR	GT	No	No
	LOG	Open	No

(b)

		SR	
		VOT	LOG
AR	GT	No	No
	LOG	Yes	Yes

(c)

Table 1: Summary of results on (a) composability, (b) effectiveness of the greedy algorithm, and (c) constant price of anarchy. (Refer to the text for the abbreviation meanings.)

Acknowledgement. We want to thank Luca Becchetti for useful ideas, hints, and discussions.

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